Using a High-Value Resistor in Triangle Comparisons of Electrical Standards

Randolph E. Elmquist, *Senior Member, IEEE,* Neil M. Zimmerman, and William H. Huber

***Abstract*—We propose an experiment with some advantages over other direct quantum metrology triangle comparisons. First, by using a cryogenic resistor that can be calibrated, the quantized Hall resistance (QHR) standard needs to be used only for short periods. Second, the experiment does not require a voltage detector. This eliminates one external source of noise and allows fast current reversals. Third, feedback that might contribute to excess noise and superconducting quantum interference device flux-jump behavior is also absent in the primary comparison system. This experiment could be run at higher currents and without supervision for extended periods of time, to benefit from statistical reduction of noise. We have developed a cryogenic current comparator for calibrating the cryogenic resistor directly against the QHR.**

***Index Terms*—Fundamental constants, Josephson effect, quantum Hall effect, resistance standards, single electron tunneling.**

I. Introduction

FUNDAMENTAL constants underlie the Josephson effect (Josephson constant K_J), the quantum Hall effect (von Klitzing constant R_K), and the precise control of electrical charge (e) by single electron tunneling (SET). New and precise experiments could improve our knowledge of these constants, for example by passing a current $I_S = f_{\mathrm{SET}} e$ through the quantized Hall resistance $R_H = R_K/i$ and comparing the resulting voltage to the Josephson voltage $V_J = n f_{\mathrm{JVS}}/K_J$. Based on Ohm's law, such a "metrology triangle" experiment would relate the ratio of the Josephson microwave frequency f_{JVS} and the SET pump cycle frequency f_{SET} to the dimensionless product

$$eR_K K_J = \frac{a f_{\mathrm{JVS}}}{f_{\mathrm{SET}}} = 2, \qquad (1)$$

where a represents a ratio of experimental integers, including the Josephson step number n, Hall plateau quantum number i, and number of electrons per pump cycle. From the 1998 CODATA recommended values of fundamental constants, measurements and theory that contribute to this combination of fundamental constants have a combined relative uncertainty of $u_r = 7.8 \times 10^{-8}$.

Several groups have proposed to measure the metrology triangle based on amplification of the SET current with a cryogenic current comparator (CCC) bridge [1]–[3]. Using a superconducting quantum interference device (SQUID) magnetic flux detector, this type of CCC bridge circuit amplifies the SET current by an integer factor of up to 10^5 and measures the amplified current in a secondary circuit using the Josephson voltage standard (JVS) and quantized Hall resistance (QHR) standard. Similar experiments based on counting electrons deposited on a cryogenic capacitor from a SET pump have reached a noise level of a few parts in a million [4]. In contrast to the direct metrology triangle which is based on Ohm's law, capacitance measurements are limited by experimental knowledge of an impedance such as a calculable capacitor value in Farads in terms of the QHR, presently known to a relative uncertainty of about 3×10^{-8}.

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R. E. Elmquist and N. M. Zimmerman are with the National Institute of Standards and Technology (NIST), Gaithersburg, MD 20899-8112 USA.

W. H. Huber is with General Electric Corporate Research and Development, Niskayuna, NY 12301 USA.

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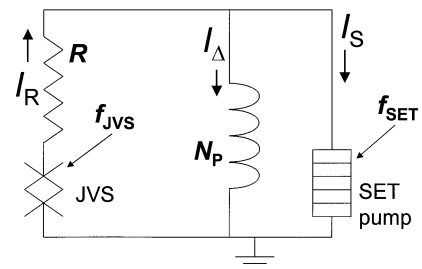

Fig. 1. Basic experimental design of the current detection circuit, using a single winding as the detection element. The current through the winding would be sensed by a SQUID magnetic flux detector.

II. Experimental Concept

Unlike the experiments above, this experiment would directly monitor a balance between two small, nearly equal currents. One current is induced by the voltage from a programmable JVS across a cryogenic resistor, and the other is generated by a SET pump device, as shown in Fig. 1. The difference I_Δ is carried by a detector winding. We would measure this small current near the balance point using a SQUID sensor in a CCC configuration, and determine the critical frequency ratio $(f_{\mathrm{SET}}/f_{\mathrm{JVS}})$ that produces null indication. One benefit of this design is that it allows the SQUID to be the only low-noise detector in the experiment.

When a CCC system is used to detect a small current, the input winding reduces the effective source resistance R_P by a factor of N_P^2 and increases the level of current seen by the detector by a factor of N_P. The rms current noise produced in the source resistance

$$\left\langle I_P^2 \right\rangle^{1/2} = \sqrt{\frac{4kT}{R_P}} \qquad (2)$$

is also amplified by the same factor. By increasing N_P, the current noise of the SQUID detector can be reduced relative to this resistor noise contribution or relative to any uncorrelated source of input noise.

The winding in Fig. 1 would also serve as the primary winding of N_P turns in a CCC bridge, for calibrating the cryogenic resistor using the QHR. Below are the requirements of the cryogenic resistor, the CCC bridge used for the resistor calibration, and the closely related detection circuit. Our intent here is to cover the areas where significant research is required for this experiment to succeed, as compared to other competing experiments [2], [3]. Progress on the research apparatus is described in Section III.

A. Cryogenic Resistor

A cryogenic resistor of value $R_P \cong 100$ MΩ at 0.3 K contributes about 0.4 $\mathrm{fA}/\sqrt{\mathrm{Hz}}$ of current noise. That corresponds to the noise of a good commercial SQUID in a CCC system with $N_P = 40\,000$, for signal frequencies at or above the $1/f$ noise corner. About this same level of current noise is produced in a circuit with a source resistance of 100 GΩ at $T = 293$ K. The advantage of a 100-MΩ cryogenic resistor is that it can be calibrated against the QHR in one step. The CCC ratio is no longer practical with higher value resistors and it would be necessary to make an intermediate calibration, introducing larger potential scaling errors.

Direct current leakage to ground has the greatest effect on resistance bridges when a leakage path occurs near the midpoint of a resistor [5]. Similarly, the time constant is largest for capacitive current near the middle of the resistor, and this time constant would be prohibitive for conventional wire-wound resistors. For a thin-film cryogenic resistor of value 100 MΩ, the time constant of the charging current is of order 1 ms.

At low temperatures $(T < 4\,\mathrm{K})$ quantum effects may begin to dominate the temperature coefficient of resistance (TCR) [6] for a thin-film resistor. The most significant contributions are due to disorder-enhanced electron-electron interactions and weak localization. These contributions depend on electron thermal and phase coherence lengths which increase at lower temperature in the resistive material and, in a thin film, scale roughly with the resistivity divided by film thickness. An increase in resistance can also result from Kondo magnetic impurity scattering and this likewise results in a more negative TCR at low temperatures.

The resistivity of the alloy also affects the length and geometrical factors required for the resistor meander pattern. We expect that the upper limit on the deposition area would be 1 cm^2, requiring a line width between 2 and 0.5 μm for 100-MΩ resistors. Alloys are required to have relatively low TCR below 4 K, moderate resistivity, and good properties as thin films. The resistance per square for these films would be of order 1 to 10 Ω.

B. CCC Bridge

This bridge would utilize a CCC with a very large turns ratio to compare a resistor of order 100 M Ω directly with the QHR $i = 2$ plateau resistance. A similar CCC bridge [7] of lower turns ratio was developed and used for calibration of cryogenic resistors soon after I. K. Harvey's first description of the CCC shielding principle. Recently, several CCC bridges have been used for comparison of high-value resistors [8], [9]. Like some of these bridges, this bridge is powered by a single ramping current source. We have constructed and tested a prototype of this CCC bridge using the design shown in Fig. 2, as described later.

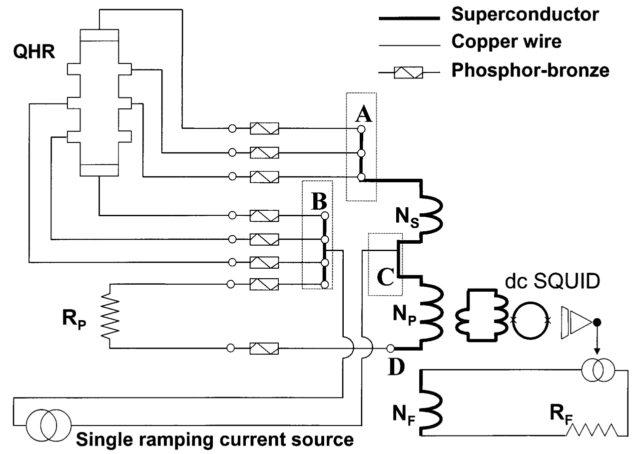

Fig. 2. Circuit design for the CCC bridge, with $R_P = 1$ MΩ (at room temperature), $N_P = 1937$, $N_S = 25$, and $N_F = 1$. Note that in the proposed experiment, the CCC ratio would be about a factor 100 higher.

This CCC bridge design allows accurate two-terminal comparisons by using multiple-series connections to the QHR [10]. With this technique, errors due to lead resistance in the lower-resistance (secondary) bridge arm can be eliminated [11], despite the lead resistance of order 10 Ω in each connection to the QHR device. The combined multiple-series voltage drop across sets of three or more such leads is about 0.3 nV at 50-μA current.

In general, the operation of this circuit is similar to that of conventional CCC bridges. The measurement design also relies on three separate equipotential junctions (labeled $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$ in Fig. 2), made with superconductor inside the CCC cryostat. These junctions allow current to split and recombine without creating voltage differences at the junction points. Two sets of three leads from a QHR standard in a separate cryostat are connected to the bridge at junction points $\mathbf{A}$ and $\mathbf{B}$. Current divides between the two arms of the bridge circuit at junction $\mathbf{B}$ and recombines at junction $\mathbf{C}$, between the superconducting primary and secondary CCC windings. The primary-arm winding senses the small current passing through the cryogenic resistor, and the remainder passes through the QHR device and winding in the secondary arm. The ampere-turns difference signal can be used to generate a compensation-feedback current. This feedback current I_F is passed through a third winding to null the SQUID output and is measured using a known sense resistor R_F, and the ratio equation

$$\frac{R_P}{R_S} = \frac{(N_P I_P + N_F I_F)}{N_S I_P} \tag{3}$$

where $R_S = R_K/2 = 12\,906.4035$ Ω, can be used to calculate the value R_P of the primary resistor.

High- and low-power loading characterization of the cryogenic resistor against the QHR will be necessary given the very small power dissipation in the SET phase of the experiment. This would relate the resistor calibration at higher currents to

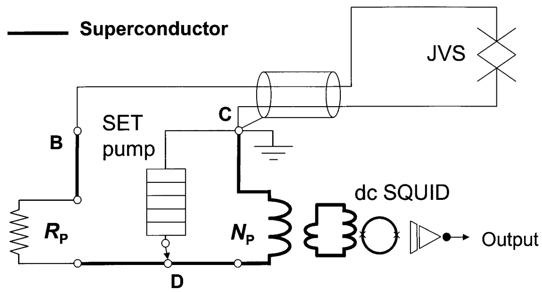

Fig. 3. Circuit design for the current detection phase of the experiment, with $R_P \cong 100\ \text{M}\Omega$. The detection winding N_P is of order 40 000 turns. Points $\mathbf{B}$, $\mathbf{C}$, and $\mathbf{D}$ shown here correspond to the same points in Fig. 2.

its value for current levels provided by the SET pump. In the case $R_P = 100\ \text{M}\Omega$, the power dissipation at moderate current levels is 25 pW for a 500-pA applied current. The maximum calibration current of about 10 nA for a 100-MΩ resistor is set by the onset of dissipation in the QHR standard.

C. Current Detection Circuit

Fig. 1 is reproduced in more detail in Fig. 3. As shown in Fig. 3, a programmable JVS system in a separate cryostat provides a voltage which determines the current through the cryogenic resistor. The QHR leads and feedback electronics are disconnected in this phase. The SET pump is connected using a point-contact cryogenic switch at point $\mathbf{D}$ in Fig. 3. This switch is present to allow for the tuning procedure [12] needed for the high accuracy of the pump current.

The SQUID detector output is measured directly during this phase of measurement. We eliminate active feedback by setting the parameters of voltage, current, and resistance such that $V_J = I_S R$. This design would run with current reversal frequencies of 1 Hz or higher with the JVS and SET pump reversed simultaneously.

III. Development

A. Detection Circuit

Computer-assisted circuit analysis and analytic modeling are useful to understand transient effects and particularly the equivalent LC (tank circuit) resonance in a large CCC winding such as that used in our detector. A large diameter winding with $N_P \geq 20\,000$ turns results in an inductance of 10 Henrys or more and stray and interwinding capacitance of order 100 to 1000 pF. Self-sustaining circuit oscillations have been observed in such CCC systems, which are usually made with low-resistance or superconducting windings. Generally, a resistance R cannot be added in parallel to a CCC winding because this would add noise to the detection circuit. In Fig. 1, the cryogenic resistor with $R \cong 100\ \text{M}\Omega$ effectively in parallel with the detector winding is an acceptable source of noise and yet will act to damp LC oscillations. In a simple equivalent circuit with N_P represented by lumped parallel inductance and capacitance ($C \cong 100$ pF) to ground, LC oscillations are thus damped with a time constant of $RC \cong 0.010$ s. However, in a more realistic model, the cryogenic resistor is less effectively coupled to the distributed LC circuit involving interwinding capacitance.

We are investigating using series resistance in damping distributed LC resonances in the CCC system. CCC detection schemes should be compatible with significant ($R > 10\ \text{k}\Omega$) distributed resistance in the primary detection winding, when the external circuit seen by the detector has a much higher impedance. This suggests that resistance-alloy wire might be superior for large CCC windings used with small currents.

B. Cryogenic Thin-Film Resistors

Thin-film resistors can be produced with precise control of the constituents, and in some alloys, the low-temperature TCR can be adjusted using appropriate minority constituents. A low TCR is highly desirable since power dissipation will result in changes in the lattice and electron temperatures, and we expect to be operating the resistor at different current levels in the two phases of this experiment. Preliminary design criteria are based on operation of the resistor at the base temperature of a ^{3}He refrigerator with the sample in liquid ^{3}He to maximize heat conduction to the environment.

We have used photolithography and electron beam co-deposition to fabricate thin film CuSi samples with various Si concentrations (0.3 ppm Fe is the major contaminant). These samples were made with line widths down to 5 μm. CuSi meander lines with moderate (200 kΩ) to low (7 kΩ) resistance have TCR near $-11\ \mu\Omega/\Omega \cdot \text{K}$ between 2 and 4 K, with an expected increase in TCR to about $-30\ \mu\Omega/\Omega \cdot \text{K}$ at 0.3 K [6]. We are also making similar CuGe samples using thermal evaporation. Samples were made of PdAu alloy by NIST colleagues in Boulder, with resistance of 100 kΩ to 30 MΩ and TCR near $-80\ \mu\Omega/\Omega \cdot \text{K}$ between 2 and 4 K. Later in the development of the experiment, we plan to use a dilution refrigerator to characterize the thin film resistors at lower temperatures.

C. 1-MΩ CCC Bridge

A bridge as pictured in Fig. 2 with $N_P = 1937$ and $N_S = 25$ was constructed and has been used to compare room temperature 1-MΩ resistance standards directly against the QHR $i = 2$ plateau, at measurement voltage of 0.74 V. Here, Johnson noise in the 1-MΩ resistor is much larger than the noise contribution of the dc SQUID. The relative type-A uncertainty for ratio measurements of 30 min duration (including reversal time), at slightly below 1×10^{-8}, corresponds to about a factor of two greater than the relative rms Johnson noise contribution of $0.18 \times 10^{-6}/\sqrt{\text{Hz}}$.

The CCC is characterized by a sensitivity of $3.8\ \mu\text{A} \cdot \text{turn}/\Phi_0$ when coupled by a five-turn flux transformer of outer diameter 45 mm. The flux transformer was constructed with 1.6 mm diameter hollow superconducting tube threaded by a 0.012 mm NbTi wire. The system has a sustained oscillation at about 20 kHz. This had little apparent effect on the measurement stability but may have contributed to the noise. The CCC outer diameter is about 0.74 times the diameter of the inner superconducting lead foil shield used to eliminate external fields, and both the CCC winding shield and the flux transformer tube are grounded.

Phosphor bronze wiring was used in the resistor leads of the CCC bridge. The two cryostat leads of approximately 3 Ω each in the primary arm are significant compared to 1 MΩ. The lead resistance values, the contact resistance of the connectors, and the resistances of external leads were measured. There are five leads connecting to the equipotential junction **B** by which any one lead resistance can be measured. Similarly, the other important lead resistance in Fig. 2 can be measured in series with one or both of the superconducting CCC windings through junctions **A** and **C**. The leads inside the cryostat were measured at several different liquid helium levels to better than 0.005 Ω. Changes in the lead resistance were of order 0.01 Ω over 40% range of liquid helium level, and were repeatable after transfer of liquid helium.

The CCC ratio results agreed very well with scaling from 10 kΩ using the best available guarded oil-type Hamon device, where the relative scaling difference was better than 2×10^{-8}. To avoid leakage errors, ground connections to case and shields should be present at the output of both the main current source and the isolated current source supplying the feedback current.

IV. Conclusion

A cryogenic resistor with $R \cong 100$ MΩ allows a null-detection measurement of the SET current, in a system that could be used with SET pumps supplying 2×10^{-12} A to 1×10^{-8} A. This type of experiment may help to reduce the number of problems associated with precision measurements of SET currents. In particular, we see two major challenges: a) the design of the low-noise primary CCC winding and SQUID detector and b) improvement in the accuracy and speed of the SET pump.

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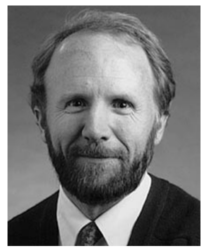

Randolph E. Elmquist (SM'98) received the Ph.D. degree in atomic physics from the University of Virginia, Charlottesville, in 1986.

In 1986, he joined the Electricity Division, National Institute of Standards and Technology, Gaithersburg, MD. Since 1995, he has headed the Electricity Division project responsible for resistance standards and the quantized Hall resistance. He has contributed to the realization of the watt in SI units, determinations of fundamental constants, and development of calculable coaxial resistance standards and cryogenic current comparators.

Dr. Elmquist is a member of the American Physical Society.

Neil M. Zimmerman was born on June 21, 1960. He received the B.S. degree in physics from Rensselaer Polytechnic Institute, Troy, NY, in 1982, and the M.S. and Ph.D. degrees in physics from Cornell University, Ithaca, NY, in 1989.

He worked as a Post-Doctoral Associate at AT&T Bell Labs from 1989 to 1992 and as a Research Physicist at the Naval Research Lab from 1992 to 1994. He is currently employed as a Research Physicist in the Fundamental Electrical Measurements Group of the National Institute of Standards and Technology, Gaithersburg, MD, where he works on applications of single-electron tunneling devices for standards of electrical capacitance or current.

William H. Huber received the B. Physics and Ph.D. degrees from the School of Physics and Astronomy, University of Minnesota, Minneapolis, in 1992 and 1999, respectively.

After finishing his degree, he was a National Research Council Postdoctoral Fellow in the Fundamental Electrical Measurements Group, National Institute of Standards and Technology (NIST), Gaithersburg, MD. His work at NIST focused on the fabrication and characterization of Single Electron Tunneling Devices for use in Electrical Metrology. In May 2001, he accepted a position at the General Electric (GE) Global Research Center, Niskayuna, NY. At GE, his research areas have included digital x-ray detectors for medical imaging and nanoparticle MRI contrast agents for molecular imaging.